

Hadron Freeze-out and QGP Hadronization

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Abstract

Abundances and m_{\perp} -spectra of strange and other hadronic particles emanating from central 158–200 A GeV reactions between nuclei are found to originate from a thermally equilibrated, deconfined source in chemical non-equilibrium. Physical freeze-out parameters and physical conditions (pressure, specific energy, entropy, and strangeness) are evaluated. Five properties of the source we obtain are as expected for direct hadron emission (hadronization) from a deconfined quark-gluon plasma phase.

1 Introduction

Quark-gluon plasma (QGP) is, by the meaning of these words, a thermally equilibrated state consisting of mobile, color charged quarks and gluons. Thermal equilibrium is established faster than the particle abundance (chemical) equilibrium and thus in general quark and gluon abundances in QGP can differ from their equilibrium Stefan-Boltzmann limit. This in turn impacts the hadronic particle production yields, and as we shall see, the chemical non-equilibrium is a key ingredient in the successful data analysis of experiments performed at the CERN-SPS in the past decade. We address here results of 200 A GeV Sulphur (S) beam interactions with laboratory stationary targets, such as Gold (Au), Tungsten (W) or Lead (Pb) nuclei and, Pb–Pb collisions at 158 A GeV. In these interactions the nominal center of momentum (CM) available energy is 8.6–9.2 GeV per participating nucleon.

Considerable refinement of data analysis has occurred since last comprehensive review of the field [1]. Our present work includes in particular the following:

1. We considered aside of strange also light quark ($q = u, d$) chemical non-equilibrium abundance [2] and introduce along with the statistical strangeness non-equilibrium parameter γ_s , its light quark analogue γ_q .
2. Coulomb distortion of the strange quark phase space has been understood [3].

Incorporating these developments, we accurately describe abundances of strange as well as non-strange hadrons, both at central rapidity and in 4π -acceptance. We are thus combining in the present analysis the strangeness diagnostic tools of dense matter with the notion of entropy enhancement in deconfined phase [4].

As particle emerge from the QGP hadronization, not only their abundance but also their spectral shape is of interest. Our analysis considers the impact of explosive radial flow on the spectra of particles at high m_{\perp} . This contributes significant information about the fireball dynamics and the possibly deconfined nature of the hadron source.

2 The Coulomb effect in the quark fireball

The diverse statistical chemical parameters that we need to consider will in a self-explanatory way be now introduced, considering the concept of strangeness balance: since strangeness can only be made and destroyed by hadronic interactions in pairs, the net abundance in the hadronic fireball must vanish. We consider a hot gas of free quarks, and evaluate the difference between strange and anti-strange quark numbers (net strangeness). The Coulomb potential originating in the initial proton abundance distorts slightly the Fermi particle distributions: strange quarks (negative charge) are attracted, while anti-strange quarks repelled. Allowing for this slowly changing potential akin to the relativistic Thomas-Fermi phase space occupancy model at finite temperature, the energy of a quark depends on both the momentum and the Coulomb potential V :

$$E_p = \sqrt{m^2 + p^2} - \frac{1}{3}V. \quad (1)$$

It is helpful to see here in first instance the potential V as a square well, Eq.(1) within the volume of interest. Counting the states in the fireball we obtain:

$$\langle N_s - N_{\bar{s}} \rangle = \int_{R_f} g_s \frac{d^3r d^3p}{(2\pi)^3} \left[\frac{1}{1 + \gamma_s^{-1} \lambda_s^{-1} e^{(\sqrt{m^2 + p^2} - \frac{1}{3}V)/T}} - \frac{1}{1 + \gamma_s^{-1} \lambda_s e^{(\sqrt{m^2 + p^2} + \frac{1}{3}V)/T}} \right]. \quad (2)$$

In Eq. (2) the subscript R_f on the spatial integral reminds us that only the classically allowed region within the fireball is covered in the integration over the level density, $g_s (= 6)$ is the quantum degeneracy of strange quarks. The magnitude of the charge of strange quarks ($Q_s/|e| = 1/3$) is shown explicitly, the Coulomb potential refers to a negative integer probe charge.

The fugacity λ_s of strange quarks enters particles and antiparticles with opposite power, while the occupancy parameter γ_s enters both term with same power. For $\gamma_s < 1$ one obtains a rather precise result for the range of parameters of interest to us (see below) considering the Boltzmann approximation:

$$\langle N_s - N_{\bar{s}} \rangle \simeq \frac{\int_{R_f} d^3r \left[\lambda_s e^{\frac{V}{3T}} - \lambda_s^{-1} e^{-\frac{V}{3T}} \right]}{\int_{R_f} d^3r} g_s \int \frac{d^3p d^3x}{(2\pi)^3} \gamma_s e^{-\sqrt{p^2 + m^2}/T}. \quad (3)$$

The meaning of the different factors is now evident. γ_s controls overall abundance of strange quark pairs, multiplying the usual Boltzmann thermal factor while λ_s controls the difference between the number of strange and non-strange quarks. Since strangeness is produced as s, \bar{s} -pair, the value of λ_s fulfills the constraint

$$\int_{R_f} d^3r \left[\lambda_s e^{\frac{V}{3T}} - \lambda_s^{-1} e^{-\frac{V}{3T}} \right] = 0, \quad (4)$$

which is satisfied exactly both in the Boltzmann limit Eq.(3), and for the exact quantum distribution Eq. (2), when:

$$\tilde{\lambda}_s \equiv \lambda_s \lambda_Q^{1/3} = 1, \quad \lambda_Q \equiv \frac{\int_{R_f} d^3r e^{\frac{V}{T}}}{\int_{R_f} d^3r}. \quad (5)$$

λ_Q is not a fugacity that can be adjusted to satisfy a chemical condition, its value is defined by the applicable Coulomb potential V . More generally, in order to account for the Coulomb effect, the quark fugacities within a deconfined region should be renamed as follows in order to absorb the Coulomb potential effect:

$$\begin{aligned} \lambda_s &\rightarrow \tilde{\lambda}_s \equiv \lambda_s \lambda_Q^{1/3}, & \lambda_d &\rightarrow \tilde{\lambda}_d \equiv \lambda_d \lambda_Q^{1/3}, \\ \lambda_u &\rightarrow \tilde{\lambda}_u \equiv \lambda_u \lambda_Q^{-2/3}, & \lambda_q &\rightarrow \tilde{\lambda}_q \equiv \sqrt{\tilde{\lambda}_u \tilde{\lambda}_d} = \lambda_q \lambda_Q^{-1/6}. \end{aligned} \quad (6)$$

Since $Q_d = Q_s = -1/3$, the first line is quite evident after the above strangeness discussion, the second follows with $Q_u = +2/3$. Note that for a negatively charged strange quark the tilded fugacity Eq. (5) contains a factor with positive power $1/3$ but the potential that enters the quantity λ_Q is negative, and thus $\tilde{\lambda}_s < \lambda_s$. Because Coulomb-effect acts in opposite way on u and d quarks, its net impact on λ_q is smaller

than on λ_s , and it also acts in the opposite way with $\tilde{\lambda}_q > \lambda_q$. To see the relevance of the tilde fugacities for light quarks, note that in order to obtain baryon density in QGP one needs to use the tilde-quark fugacity to account for the Coulomb potential influence on the phase space.

It is somewhat unexpected that for the Pb–Pb fireball the Coulomb effect is at all relevant. Recall that for a uniform charge distribution within a radius R_f of charge Z_f :

$$V = \begin{cases} -\frac{3}{2} \frac{Z_f e^2}{R_f} \left[1 - \frac{1}{3} \left(\frac{r}{R_f} \right)^2 \right], & \text{for } r < R_f; \\ -\frac{Z_f e^2}{r}, & \text{for } r > R_f. \end{cases} \quad (7)$$

Choosing $R_f = 8$ fm, $T = 140$ MeV, $m_s = 200$ MeV (value of γ_s is practically irrelevant) we find as solution of Eq. (2) for $\langle N_s - N_{\bar{s}} \rangle = 0$ for $Z_f = 150$ the value $\lambda_s = 1.10$ (precisely: 1.0983; $\lambda_s = 1.10$ corresponds to $R_f = 7.87$ fm). We will see that both this values within the experimental precision arise from study of particle abundances. For the S–W/Pb reactions this Coulomb effect is practically negligible.

In the past we (and others) have disregarded in the description of the hadronic final state abundances the electrical charges and interactions of the produced hadrons. This is a correct way to analyze the chemical properties since, as already mentioned, the quantity λ_Q is not a new fugacity: conservation of flavor already assures charge conservation in chemical hadronic reactions, and use of λ_i , $i = u, d, s$ exhausts all available chemical balance conditions for the abundances of hadronic particles. As shown here, the Coulomb deformation of the phase space in the QGP fireball makes it necessary to rethink the implications that the final state particle measured fugacities have on the fireball properties.

3 Freeze-out of hadrons

The production of hadrons from a QGP fireball occurs mainly by way of quark coalescence and gluon fragmentation, and there can be some quark fragmentation as well. We will explicitly consider the recombination of quarks, but implicitly the gluon fragmentation is accounted for by our allowance for chemical nonequilibrium. The relative number of primary particles freezing out from a source is obtained noting that the fugacity and phase space occupancy of a composite hadronic particle can be expressed by its constituents and that the probability to find all j -components contained within the i -th emitted particle is:

$$N_i \propto \prod_{j \in i} \gamma_j \lambda_j e^{-E_j/T}, \quad \lambda_i = \prod_{j \in i} \lambda_j, \quad \gamma_i = \prod_{j \in i} \gamma_j. \quad (8)$$

Experimental data with full phase space coverage, or central rapidity region $|y - y_{CM}| < 0.5$, for $m_\perp > 1.5$ GeV are considered; recall that the energy of a hadron ‘ i ’ is expressed by the spectral parameters m_\perp and y as follows,

$$E_i = \sum E_j, \quad E_i = \sqrt{m_i^2 + p^2} = \sqrt{m_i^2 + p_\perp^2} \cosh(y - y_{CM}),$$

where y_{CM} is the center of momentum rapidity of the fireball formed by the colliding nuclei.

The yield of particles is controlled by the freeze-out temperature T_f . This freeze-out temperature is different from the m_\perp -spectral temperature T_\perp , which also comprises the effect of collective matter flows originating in the explosive disintegration driven by the internal pressure of compressed hadronic matter. In order to model the flow and freeze-out of the fireball surface, one in general needs several new implicit and/or explicit parameters. We therefore will make an effort to choose experimental variables (compatible particle ratios) which are largely flow independent. This approach also diminishes the influence of heavy resonance population — we include in Eq.(8) hadronic states up to $M = 2$ GeV, and also include quantum statistical corrections, allowing for first Bose and Fermi distribution corrections in the phase space content. It is hard to check if indeed we succeeded in eliminating the uncertainty about high mass hadron populations. As we shall see comparing descriptions which exclude flow with those that include it, our approach is indeed largely flow-insensitive.

We consider here a simple radial flow model, with freeze-out in CM frame at constant laboratory time, implying that causally disconnected domains of the dense matter fireball are synchronized at the instant of the collision. Within this approach [5], the spectra and thus also multiplicities of particles emitted are obtained replacing the Boltzmann exponential factor in Eq.(8),

$$e^{-E_j/T} \rightarrow \frac{1}{2\pi} \int d\Omega_v \gamma_v (1 + \vec{v}_c \cdot \vec{p}_j/E_j) e^{-\frac{\gamma_v E_j}{T} (1 + \vec{v}_c \cdot \vec{p}_j/E_j)}, \quad (9)$$

where as usual $\gamma_v = 1/\sqrt{1 - \vec{v}_c^2}$. Eq. (9) can be intuitively obtained by a Lorentz transformation between an observer on the surface of the fireball, and one at rest in the general CM (laboratory) frame. One common feature of all flow scenarios is that, at sufficiently high m_\perp , the spectral temperature (inverse slope) T_\perp can be derived from the freeze-out temperature T_f with the help of the Doppler formula:

$$T_\perp = T_f \gamma_v (1 + v_c). \quad (10)$$

In actual numerical work, we proceed as follows to account for the Doppler effect: for a given pair of values T_f and v_c , the resulting m_\perp particle spectrum is obtained and analyzed using the spectral shape and procedure employed for the particular collision system by the experimental groups, yielding the theoretical inverse slope ‘temperature’ T_\perp^j .

Once the parameters T_f , λ_q , λ_s , γ_q , γ_s and v_c have been determined studying available particle yields, and m_\perp slopes, the entire phase space of particles produced is fully characterized within our elaborate statistical model. Our model is in fact just an elaboration of the original Fermi model [6], in fact all we do is to allow hadronic particles to be produced in the manner dictated by the phase space size of valance quarks. With the full understanding of the phase space of all hadrons, we can evaluate the physical properties of the system at freeze-out, such as, *e.g.*, energy and entropy per baryon, strangeness content.

4 Results of data analysis

As noted our analysis requires that we form particle abundance ratios between what we call compatible hadrons. We considered for S–Au/W/Pb reactions 18 data points listed in table 1 (of which three comprise results with Ω ’s). For Pb–Pb we address here 15 presently available particle yield ratios listed in table 2 (of which four comprise the Ω ’s). We believe to have included in our discussion most if not all particle multiplicity results available presently.

The theoretical particle yield results shown columns in tables 1 and 2 are obtained looking for a set of physical parameters which will minimize the difference between theory and experiment. The resulting total error of the ratios R is shown at the bottom of tables 1 and 2:

$$\chi_T^2 = \sum_j \left(\frac{R_{\text{th}}^j - R_{\text{exp}}^j}{\Delta R_{\text{exp}}^j} \right)^2. \quad (11)$$

It is a non-trivial matter to determine the confidence level that goes with the different data analysis approaches since some of the results considered are partially redundant, and a few data points can be obtained from others by algebraic relations arising in terms of their theoretical definitions; there are two types of such relations:

$$\frac{\Omega + \bar{\Omega}}{\Xi + \bar{\Xi}} = \frac{\bar{\Omega}}{\bar{\Xi}} \cdot \frac{1 + \bar{\Omega}/\Omega}{1 + \bar{\Xi}/\Xi}, \quad \frac{\bar{\Lambda}}{\Lambda} = \frac{\bar{\Lambda}}{\bar{\Xi}} \cdot \frac{\bar{\Xi}}{\Lambda}. \quad (12)$$

However, due to smallness of the total error found for some of the approaches it is clear without detailed analysis that only these are statistically significant.

In addition to the abundance data, we also explored the transverse mass m_\perp -spectra when the collective flow velocity was allowed in the description, and the bottom line of tables 1 and 2 in columns D_v , F_v includes in these cases the error found in the inverse slope parameter of the spectra. The procedure we used is as follows: since within the error the high m_\perp strange (anti)baryon inverse slopes are within error,

Table 1: Particle ratios studied in our analysis for S–W/Pb/Au reactions: experimental results with references and kinematic cuts are given, followed by columns showing results for the different strategies of analysis B–F. Asterisk * means a predicted result (corresponding data is not fitted). Subscript s implies forced strangeness conservation, subscript v implies inclusion of collective flow. The experimental results considered are from:

¹ S. Abatzis *et al.*, WA85 Collaboration, *Heavy Ion Physics* **4**, 79 (1996).

² S. Abatzis *et al.*, WA85 Collaboration, *Phys. Lett. B* **347**, 158 (1995).

³ S. Abatzis *et al.*, WA85 Collaboration, *Phys. Lett. B* **376**, 251 (1996).

⁴ I.G. Bearden *et al.*, NA44 Collaboration, *Phys. Rev. C* **57**, 837 (1998).

⁵ D. Röhrich for the NA35 Collaboration, *Heavy Ion Physics* **4**, 71 (1996).

⁶ S–Ag value adopted here: T. Alber *et al.*, NA35 Collaboration, *Eur. Phys. J. C* **2**, 643 (1998).

⁷ A. Iyono *et al.*, EMU05 Collaboration, *Nucl. Phys. A* **544**, 455c (1992) and Y. Takahashi *et al.*, EMU05 Collaboration, private communication.

Ratios	Ref.	Cuts [GeV]	Exp.Data	B	C	D	D _s	F	D _v	F _v
Ξ/Λ	1	$1.2 < p_{\perp} < 3$	0.097 ± 0.006	0.16	0.11	0.099	0.11	0.10	0.11	0.11
$\bar{\Xi}/\bar{\Lambda}$	1	$1.2 < p_{\perp} < 3$	0.23 ± 0.02	0.38	0.23	0.22	0.18	0.22	0.23	0.22
$\bar{\Lambda}/\Lambda$	1	$1.2 < p_{\perp} < 3$	0.196 ± 0.011	0.20	0.20	0.203	0.20	0.20	0.20	0.20
$\bar{\Xi}/\Xi$	1	$1.2 < p_{\perp} < 3$	0.47 ± 0.06	0.48	0.44	0.45	0.33	0.44	0.44	0.43
$\bar{\Omega}/\Omega$	2	$p_{\perp} > 1.6$	0.57 ± 0.41	1.18*	0.96*	1.01*	0.55*	0.98	1.09*	1.05*
$\frac{\Omega+\bar{\Omega}}{\Xi+\bar{\Xi}}$	2	$p_{\perp} > 1.6$	0.80 ± 0.40	0.27*	0.17*	0.16*	0.16*	0.16	0.13*	0.13*
K^+/K^-	1	$p_{\perp} > 0.9$	1.67 ± 0.15	2.06	1.78	1.82	1.43	1.80	1.77	1.75
K_s^0/Λ	3	$p_{\perp} > 1$	1.43 ± 0.10	1.56	1.64	1.41	1.25	1.41	1.38	1.39
$K_s^0/\bar{\Lambda}$	3	$p_{\perp} > 1$	6.45 ± 0.61	7.79	8.02	6.96	6.18	6.96	6.81	6.86
K_s^0/Λ	1	$m_{\perp} > 1.9$	0.22 ± 0.02	0.26	0.28	0.24	0.24	0.24	0.24	0.24
$K_s^0/\bar{\Lambda}$	1	$m_{\perp} > 1.9$	0.87 ± 0.09	1.30	1.38	1.15	1.20	1.16	1.18	1.17
Ξ/Λ	1	$m_{\perp} > 1.9$	0.17 ± 0.01	0.27	0.18	0.17	0.18	0.17	0.16	0.17
$\bar{\Xi}/\bar{\Lambda}$	1	$m_{\perp} > 1.9$	0.38 ± 0.04	0.64	0.38	0.38	0.30	0.37	0.35	0.35
$\frac{\Omega+\bar{\Omega}}{\Xi+\bar{\Xi}}$	1	$m_{\perp} > 2.3$	1.7 ± 0.9	0.98*	0.59*	0.58*	0.52*	0.58	0.72*	0.75*
p/\bar{p}	4	Mid-rapidity	11 ± 2	11.2	10.1	10.6	7.96	10.5	10.6	10.4
$\bar{\Lambda}/\bar{p}$	5	4π	1.2 ± 0.3	2.50	1.47	1.44	1.15	1.43	1.58	1.66
$\frac{h^-}{p-\bar{p}}$	6	4π	4.3 ± 0.3	4.4	4.2	4.1	3.6	4.1	4.2	4.2
$\frac{h^+-h^-}{h^++h^-}$	7	4π	0.124 ± 0.014	0.11	0.10	0.103	0.09	0.10	0.12	0.12
χ^2_{T}				264	30	6.5	38	12	6.2	11

overlapping we decided to consider just one ‘mean’ experimental value $T_{\perp} = 235 \pm 10$ for S–induced reactions and $T_{\perp} = 265 \pm 15$ for Pb–induced reactions. Thus we add one experimental value and one parameter, without changing the number of degrees of freedom. Once we find values of T_f and v_c , we evaluate the slopes of the theoretical spectra. The resulting theoretical T_{th}^j values are in remarkable agreement with experimental T_{\perp}^j , well beyond what we expected, as is shown in table 3. An exception is the fully strange $\Omega + \bar{\Omega}$ spectrum. We note in passing that when v_c was introduced we found little additional correlation between now 6 theoretical parameters. The collective flow velocity is a truly new degree of freedom and it helps to attain a more consistent description of the experimental data available.

Although it is clear that one should be using a full-fledged model such as D_v , we address also cases B and C. The reason for this arises from our desire to demonstrate the empirical need for chemical non-equilibrium: in the approach B, complete chemical equilibrium $\gamma_i = 1$ is assumed. As we see in tables 1 and 2 this approach has rather large error. Despite this the results in column B in tables 1 and 2 are often compared favorably with experiment, indeed this result can be presented quite convincingly on a logarithmic scale. Yet as we see the disagreement between theory and experiment is quite forbidding. With this remark we wish to demonstrate the need for comprehensive and precisely measured hadron abundance data sample, including abundances of multi-strange antibaryons, which were already 20 years ago identified as the hadronic signals

Table 2: Particle ratios studied in our analysis for Pb–Pb reactions: experimental results with references and kinematic cuts are given, followed by columns showing results for the different strategies of analysis B–F. Asterisk * means a predicted result (corresponding data is not fitted or not available). Subscript s implies forced strangeness conservation, subscript v implies inclusion of collective flow. The experimental results considered are from:

- ¹ I. Králik, for the WA97 Collaboration, *Nucl. Phys. A* **638**, 115, (1998).
² G.J. Odyniec, for the NA49 Collaboration, *J. Phys. G* **23**, 1827 (1997).
³ P.G. Jones, for the NA49 Collaboration, *Nucl. Phys. A* **610**, 188c (1996).
⁴ F. Pühlhofer, for the NA49 Collaboration, *Nucl. Phys. A* **638**, 431, (1998).
⁵ C. Bormann, for the NA49 Collaboration, *J. Phys. G* **23**, 1817 (1997).
⁶ G.J. Odyniec, *Nucl. Phys. A* **638**, 135, (1998).
⁷ D. Röhrig, for the NA49 Collaboration, “Recent results from NA49 experiment on Pb–Pb collisions at 158 A GeV”, see Fig. 4, in proc. of EPS-HEP Conference, Jerusalem, Aug. 19-26, 1997.
⁸ A.K. Holme, for the WA97 Collaboration, *J. Phys. G* **23**, 1851 (1997).

Ratios	Ref.	Cuts [GeV]	Exp.Data	B	C	D	D _s	F	D _v	F _v
Ξ/Λ	1	$p_{\perp} > 0.7$	0.099 ± 0.008	0.138	0.093	0.095	0.098	0.107	0.102	0.110
$\Xi/\bar{\Lambda}$	1	$p_{\perp} > 0.7$	0.203 ± 0.024	0.322	0.198	0.206	0.215	0.216	0.210	0.195
$\bar{\Lambda}/\Lambda$	1	$p_{\perp} > 0.7$	0.124 ± 0.013	0.100	0.121	0.120	0.119	0.121	0.123	0.128
Ξ/Ξ	1	$p_{\perp} > 0.7$	0.255 ± 0.025	0.232	0.258	0.260	0.263	0.246	0.252	0.225
$\frac{(\Xi+\bar{\Xi})}{(\Lambda+\bar{\Lambda})}$	2	$p_{\perp} > 1.$	0.13 ± 0.03	0.169	0.114	0.118	0.122	0.120	0.123	0.121
K_s^0/ϕ	3,4		11.9 ± 1.5	6.3	10.4	9.89	9.69	16.1	12.9	15.1
K^+/K^-	5		1.80 ± 0.10	1.96	1.75	1.76	1.73	1.62	1.87	1.56
p/\bar{p}	6		$18.1 \pm 4.$	22.0	17.1	17.3	17.9	16.7	17.4	15.3
$\bar{\Lambda}/\bar{p}$	7		$3. \pm 1.$	3.02	2.91	2.68	3.45	0.65	2.02	1.29
K_s^0/B	3		0.183 ± 0.027	0.305	0.224	0.194	0.167	0.242	0.201	0.281
h^-/B	3		1.83 ± 0.2	1.47	1.59	1.80	1.86	1.27	1.83	1.55
Ω/Ξ	1	$p_{\perp} > 0.7$	0.192 ± 0.024	0.119*	0.080*	0.078*	0.080*	0.192	0.077*	0.190
$\bar{\Omega}/\bar{\Xi}$	8	$p_{\perp} > 0.7$	0.27 ± 0.06	0.28*	0.17*	0.17*	0.18*	0.40	0.18*	0.40
$\bar{\Omega}/\Omega$	1	$p_{\perp} > 0.7$	0.38 ± 0.10	0.55*	0.56*	0.57*	0.59*	0.51	0.60*	0.47
$\frac{(\Omega+\bar{\Omega})}{(\Xi+\bar{\Xi})}$	8	$p_{\perp} > 0.7$	0.20 ± 0.03	0.15*	0.10*	0.10*	0.10*	0.23	0.10*	0.23
χ^2_T				88	24	1.6	2.7	19	1.5	18

Table 3: Particle spectra inverse slopes: theoretical values T_{th} are obtained imitating the experimental procedure from the T_f, v_c -parameters. Top portion: S–W experimental T_{\perp} from experiment WA85 for kaons, lambdas and cascades; bottom portion: experimental Pb–Pb T_{\perp} from experiment NA49 for kaons and from experiment WA97 for baryons. The experimental results are from:

- D. Evans, for the WA85-collaboration, *APH N.S., Heavy Ion Physics* **4**, 79 (1996).
E. Andersen *et al.*, WA97-collaboration, *Phys. Lett. B* **433**, 209, (1998).
S. Margetis, for the NA49-collaboration, *J. Physics G, Nucl. and Part. Phys.* **25**, 189 (1999).

	T^{K^0}	T^{Λ}	$T^{\bar{\Lambda}}$	T^{Ξ}	$T^{\bar{\Xi}}$	$T^{\Omega+\bar{\Omega}}$
T_{\perp} [MeV]	219 ± 5	233 ± 3	232 ± 7	244 ± 12	238 ± 16	—
T_{th} [MeV]	215	236	236	246	246	260
T_{\perp} [MeV]	223 ± 13	291 ± 18	280 ± 20	289 ± 12	269 ± 22	237 ± 24
T_{th} [MeV]	241	280	280	298	298	335

of QGP phenomena [7]. The strange antibaryon enhancement reported by the experiment WA97 fully confirms the role played by these particles [8].

In the approach C, we introduce strangeness chemical non-equilibrium [9], *i.e.*, we also vary γ_s , keeping

Table 4: Statistical parameters which best describe the experimental S–Au/W/Pb results shown in table 1. Asterisk (*) means a fixed (input) value or result of a constraint. In approaches B to D, particle abundance ratios comprising Ω are not considered. In case D_s strangeness conservation in the particle yields was enforced. In case F the three data-points with Ω are considered. Lower index v implies that radial collective flow velocity has been considered.

S–W	T_f [MeV]	λ_q	λ_s	γ_s/γ_q	γ_q	v_c	χ^2_T
B	144 ± 2	1.53 ± 0.02	0.97 ± 0.02	1*	1*	0*	264
C	147 ± 2	1.49 ± 0.02	1.01 ± 0.02	0.62 ± 0.02	1*	0*	30
D	143 ± 3	1.50 ± 0.02	1.00 ± 0.02	0.60 ± 0.02	1.22 ± 0.06	0*	6.5
D _s	153 ± 3	1.42 ± 0.02	$1.10^* \pm 0.02$	0.56 ± 0.02	1.26 ± 0.06	0*	38
F	144 ± 3	1.49 ± 0.02	1.00 ± 0.02	0.60 ± 0.02	1.22 ± 0.06	0*	12
D _v	144 ± 2	1.51 ± 0.02	1.00 ± 0.02	0.69 ± 0.03	1.41 ± 0.08	0.49 ± 0.02	6.2
F _v	145 ± 2	1.50 ± 0.02	0.99 ± 0.02	0.69 ± 0.03	1.43 ± 0.08	0.50 ± 0.02	11

Table 5: Statistical parameters which best describe the experimental Pb–Pb results shown in table 2. Asterisk (*) means a fixed (input) value, or result of a constraint. In approaches B to D, particle abundance ratios comprising Ω are not considered. In case D_s strangeness conservation in the particle yields was enforced. In case F the four data-points with Ω are considered. Lower index v implies that radial flow velocity has been considered.

Pb–Pb	T_f [MeV]	λ_q	λ_s	γ_s/γ_q	γ_q	v_c	χ^2_T
B	142 ± 3	1.70 ± 0.03	1.10 ± 0.02	1*	1*	0*	88
C	144 ± 4	1.62 ± 0.03	1.10 ± 0.02	0.63 ± 0.04	1*	0*	24
D	134 ± 3	1.62 ± 0.03	1.10 ± 0.02	0.69 ± 0.08	1.84 ± 0.30	0*	1.6
D _s	133 ± 3	1.63 ± 0.03	$1.09^* \pm 0.02$	0.72 ± 0.12	2.75 ± 0.35	0*	2.7
F	334 ± 18	1.61 ± 0.03	1.12 ± 0.02	0.50 ± 0.01	0.18 ± 0.02	0*	19
D _v	144 ± 2	1.60 ± 0.02	1.10 ± 0.02	0.86 ± 0.03	1.72 ± 0.08	0.58 ± 0.02	1.5
F _v	328 ± 17	1.59 ± 0.03	1.13 ± 0.02	0.51 ± 0.03	0.34 ± 0.13	0.38 ± 0.31	18

$\gamma_q = 1$. A nearly valid experimental data description is now possible, and indeed, when the error bars were smaller this was a satisfactory approach adapted in many studies [1]. However, only the possibility of light quark non-equilibrium in fit D produces a statistically significant data description.

It is interesting to note that a significant degradation of χ^2_T occurs, especially in the Pb–Pb data, when we require in column F that the particle ratios comprising Ω and $\bar{\Omega}$ -particles are also described. We thus conclude that a large fraction of these particles must be made in processes that are not considered in the present model.

Another notable study case is shown in column D_s, with strangeness conservation enforced. Remarkably, the S–W data, table 1, do not like this constraint. A possible explanation is that for S-induced reactions, the particle abundances are obtained at relatively high p_\perp . Thus only a small fraction of all strange particles is actually observed, and therefore the overall strangeness is hard to balance. Similar conclusion results also when radial flow is explicitly allowed for, with a significant unbalanced strangeness fractions remaining, as we shall discuss below. On the other hand, this constraint is relatively easily satisfied for the Pb–Pb collision results, table 2, where a much greater proportion of all strange particles is actually experimentally detected.

The statistical parameters associated with the particle abundances described above are shown in the table 4 for S–W/Pb and in table 5 for Pb–Pb reactions. The errors of the statistical parameters shown are those provided by the program MINUIT96.03 from CERN program library. When the theory describes the

Table 6: T_f and physical properties (specific energy, entropy, anti-strangeness, net strangeness, pressure and volume) of the full hadron phase space characterized by the statistical parameters given in table 4 for the reactions S–Au/W/Pb. Asterisk * means fixed input.

S–W	T_f [MeV]	E_f/B	S_f/B	\bar{s}_f/B	$(\bar{s}_f - s_f)/B$	P_f [GeV/fm ³]
B	144 ± 2	8.9 ± 0.5	50 ± 3	1.66 ± 0.06	0.44 ± 0.02	0.056 ± 0.005
C	147 ± 2	9.3 ± 0.5	49 ± 3	1.05 ± 0.05	0.23 ± 0.02	0.059 ± 0.005
D	143 ± 3	9.1 ± 0.5	48 ± 3	0.91 ± 0.04	0.20 ± 0.02	0.082 ± 0.006
D_s	153 ± 3	8.9 ± 0.5	45 ± 3	0.76 ± 0.04	0*	0.133 ± 0.008
F	144 ± 2	9.1 ± 0.5	48 ± 3	0.91 ± 0.05	0.20 ± 0.02	0.082 ± 0.006
D_v	144 ± 2	8.2 ± 0.5	44 ± 3	0.72 ± 0.04	0.18 ± 0.02	0.124 ± 0.007
F_v	145 ± 2	8.2 ± 0.5	44 ± 3	0.73 ± 0.05	0.17 ± 0.02	0.123 ± 0.007

data well, this is a one standard deviation error in theoretical parameters arising from the experimental measurement error. In tables 4 and 5 in cases in which $\gamma_q \neq 1$ we present the ratio γ_s/γ_q , which corresponds approximately to the parameter γ_s in the data studies in which $\gamma_q = 1$ has been assumed. It is notable that whenever we allow phase space occupancy to vary from the equilibrium, a significant deviation is found. In the S–W case, table 4, there is a 25% excess in the light quark occupancy, while strange quarks are 25% below equilibrium. In Pb–Pb case, table 5, the ratio of the nonequilibrium parameters $\gamma_s/\gamma_q \simeq 0.7$ also varies little (excluding the failing cases F with $\Omega, \bar{\Omega}$ data), though the individual values γ_s, γ_q can change significantly, even between the high confidence cases.

We note that in the Pb–Pb reaction, table 5, $\gamma_s > 1$. This is an important finding, since an explanation of this effect involves formation prior to freeze-out in the matter at high density of near chemical equilibrium, $\gamma_s(t < t_f) \simeq 1$. The ongoing rapid expansion (note that the collective velocity at freeze-out is found to be $1/\sqrt{3}$) preserves this high strangeness yield, and thus we find the result $\gamma_s > 1$. In other words the strangeness production freeze-out temperature $T_s > T_f$. Thus the strangeness equilibration time is proven implicitly to be of magnitude expected in earlier studies of the QGP processes [10]. It is hard, if not really impossible, to arrive at this result without the QGP hypothesis. Moreover, inspecting figure 38 in [10] we see that the yield of strangeness we expect from the kinetic theory in QGP is at the level of 0.75 per baryon, the level we indeed will determine below.

Another notable results is $\tilde{\lambda}_s \simeq \lambda_s \simeq 1.0$ in the S–Au/W/Pb case, see table 4, and $\lambda_s \simeq 1.1$ in the Pb–Pb case, see table 5, implying here $\tilde{\lambda}_s = 1$, see section 2. We see clearly for both S- and Pb-induced reactions a value of λ_s , characteristic for a source of freely movable strange quarks with balancing strangeness.

5 Physical properties of the fireball at chemical freeze-out

Given the precise statistical information about the properties of the hadron phase space, we can determine the physical properties of the hadronic particles, see tables 6 and 7. We show for the same study cases B–F, along with their temperature, the specific energy and entropy content per baryon, and specific anti-strangeness content, along with specific strangeness asymmetry, and finally pressure at freeze-out. We note that it is improper in general to refer to these properties as those of a ‘hadronic gas’ formed in nuclear collisions, as the particles considered may be emitted in sequence from a deconfined source, and thus there may never be a evolution stage corresponding to a hadron gas phase. However, the properties presented are those carried away by the emitted particles, and thus characterize the properties of their source.

The energy per baryon seen in the emitted hadrons is, within error, equal to the available specific energy of the collision (8.6 GeV for Pb–Pb and about 8.8–8.9 GeV for S–Au/W/Pb). This implies that the fraction of energy left in the central fireball must be the same as the fraction of baryon number. We further note that hadrons emitted at freeze-out carry away a specific \bar{s} content which is determined to be 0.72 ± 0.04 in S–Au/W/Pb case and 0.78 ± 0.04 for the Pb–Pb collisions (cases D_v). Here we see the most significant

Table 7: T_f and physical properties (specific energy, entropy, anti-strangeness, net strangeness, pressure and volume) of the full hadron phase space characterized by the statistical parameters given in table 5 for the reactions Pb–Pb. Asterisk * means fixed input.

Pb–Pb	T_f [MeV]	E_f/B	S_f/B	\bar{s}_f/B	$(\bar{s}_f - s_f)/B$	P_f [GeV/fm ³]
B	142 ± 3	7.1 ± 0.5	41 ± 3	1.02 ± 0.05	0.21 ± 0.02	0.053 ± 0.005
C	144 ± 4	7.7 ± 0.5	42 ± 3	0.70 ± 0.05	0.14 ± 0.02	0.053 ± 0.005
D	134 ± 3	8.3 ± 0.5	47 ± 3	0.61 ± 0.04	0.08 ± 0.01	0.185 ± 0.012
D_s	133 ± 3	8.7 ± 0.5	48 ± 3	0.51 ± 0.04	0*	0.687 ± 0.030
F	334 ± 18	9.8 ± 0.5	24 ± 2	0.78 ± 0.05	0.06 ± 0.01	1.64 ± 0.06
D_v	144 ± 2	7.0 ± 0.5	38 ± 3	0.78 ± 0.04	0.01 ± 0.01	0.247 ± 0.007
F_v	328 ± 17	11.2 ± 1.5	28 ± 3	0.90 ± 0.05	0.09 ± 0.02	1.40 ± 0.06

impact of flow, as without it the specific strangeness content seemed to diminish as we moved to the larger collision system. We have already alluded repeatedly to the fact that for S–Au/W/Pb case the balance of strangeness is not seen in the particles observed experimentally. The asymmetry is 18%, with the excess emission of \bar{s} containing hadrons at high $p_\perp > 1$ GeV. In the Pb–Pb data this effect disappears, perhaps since the p_\perp lower cut-off is smaller. One could also imagine that longitudinal flow which is stronger in S–Au/W/Pb is responsible for this effect.

The small reduction of the specific entropy in Pb–Pb compared to the lighter S–Au/W/Pb is driven by the greater baryon stopping in the larger system, also seen in the smaller energy per baryon content. Both systems freeze out at $E/S = 0.185$ GeV (energy per unit of entropy). Aside of T_f , this is a second universality feature at hadronization of both systems. The overall high specific entropy content agrees well with the entropy content evaluation made earlier [4] for the S–W case. This is so because the strange particle data are indeed fully consistent within the chemical-nonequilibrium description with the 4π total particle multiplicity results.

6 Current status and conclusions

We have presented detailed analysis of hadron abundances observed in central S–Au/W/Pb 200 A GeV and Pb–Pb 158 A GeV interactions within thermal equilibrium and chemical non-equilibrium phase space model of strange and non-strange hadronic particles. In the analysis of the freeze-out the structure of the particle source was irrelevant. However, the results that we found for the statistical parameters point rather clearly towards a deconfined QGP source, with quark abundances near but not at chemical equilibrium: statistical parameters obtained characterize a strange particle source which, both for S–Au/W/Pb and for Pb–Pb case, when allowing for Coulomb deformation of the strange and anti-strange quarks, is exactly symmetric between s and \bar{s} quark carriers, as is natural for a deconfined state.

There are similarities in freeze-out properties seen comparing results presented in tables 6 and 7 which suggest universality in the extensive physical properties of the two freeze-out systems [3]. Despite considerably varying statistical parameters we obtain similar physical conditions such as T_f and E/S corresponding possibly to the common physical properties of QGP at its breakup into hadrons. The precision of the data description we have reached, see tables 1–3 strongly suggests that despite its simplicity the model we developed to analyze experimental data provides a reliable image of the hadron production processes. When flow is allowed for the freeze-out temperature is identical in both physical systems considered, even though, *e.g.*, the baryochemical potential $\mu_B = 3T \ln \lambda_q$ and other physical parameters found are slightly different. It is worth to restate the values we obtained, case D_v (6 parameter description, no Ω , $\bar{\Omega}$):

$$T_f = 144 \pm 2 \text{ MeV}, \quad \mu_B = 178 \pm 5 \text{ MeV}, \quad \text{for S–Pb/W},$$

$$T_f = 144 \pm 2 \text{ MeV}, \quad \mu_B = 203 \pm 5 \text{ MeV}, \quad \text{for Pb–Pb},$$

Even though there is still considerable uncertainty about other freeze-out flow effects, such as longitudinal flow (memory of the collision axis), the level of consistency and quality of agreement between a wide range of experimental data and our chemical non-equilibrium, thermal equilibrium statistical model suggests that, for the observables considered, these effects do not matter.

The key results we found analyzing experimental data are:

1. $\tilde{\lambda}_s = 1$ for S and Pb collisions ;
2. $\gamma_s^{\text{Pb}} > 1$, $\gamma_q > 1$;
3. $v_c^{\text{Pb}} = 1/\sqrt{3}$
4. $S/B \simeq 40$;
5. $\bar{s}/B \simeq 0.75$;

are in remarkable agreement with the properties of a deconfined QGP source hadronizing without chemical re-equilibration. The only natural interpretation of our findings is thus that hadronic particles seen at 158–200 GeV A nuclear collisions at CERN-SPS are emerging directly from a deconfined QGP state and do not undergo a chemical re-equilibration after they have been produced.

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